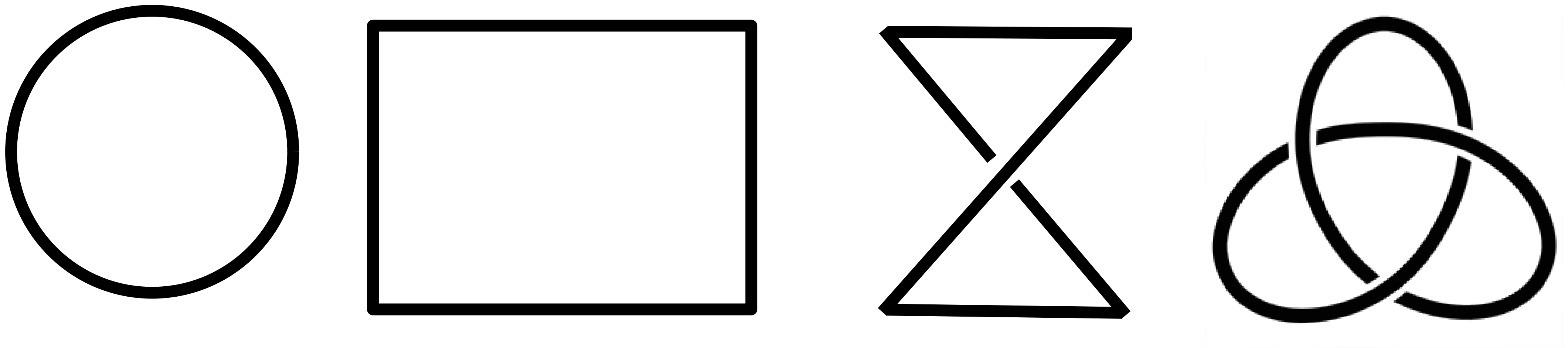
Quantum Computing (with knots)

Henrique Ennes
QuantAzur Days
24/10/2025

Knots

Knots are embeddings of the circle in $\mathbb{R}^3$.



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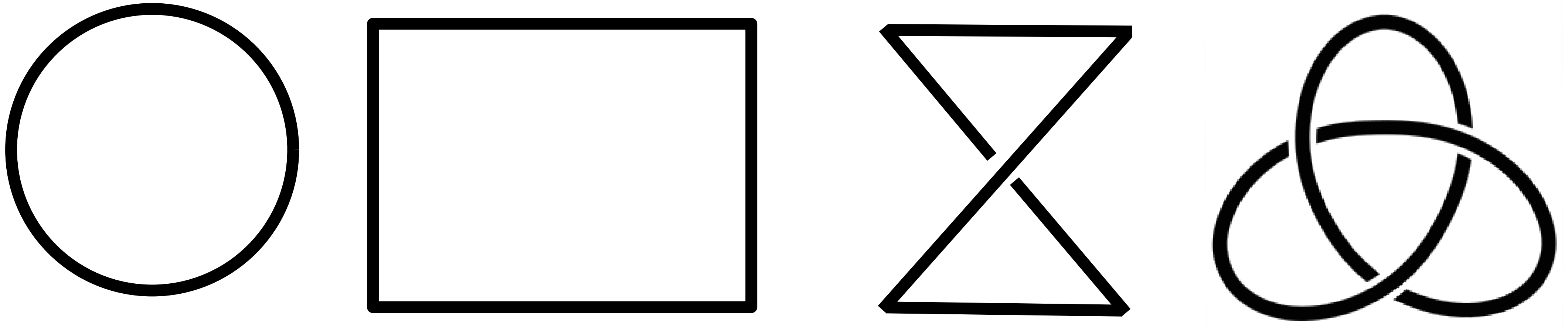


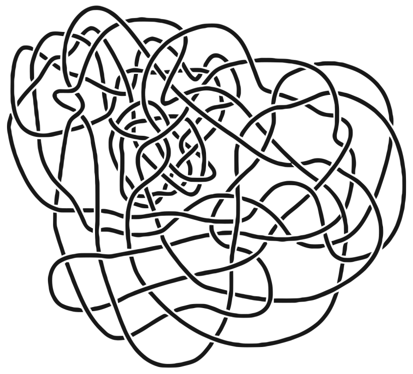
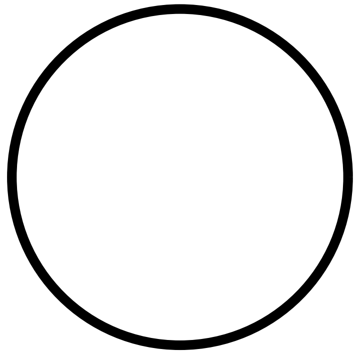
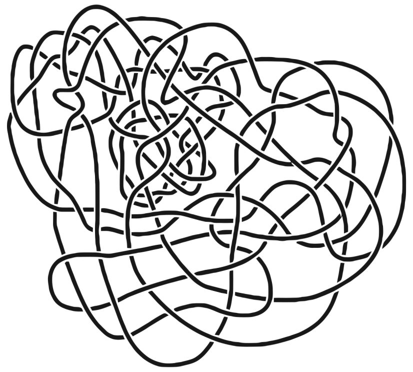
Two knots will be called *isotopic* if we can bring one to the other without tearing them apart.

 ISOTOPIC

 NON-ISOTOPIC

Knots

The problem of telling isotopic knots apart is computationally very hard.

PROBLEM			STATUS
	“ = ”		$NP \cap coNP$
	“ = ”		DECIDABLE

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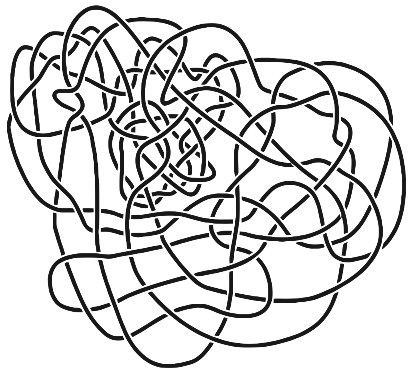
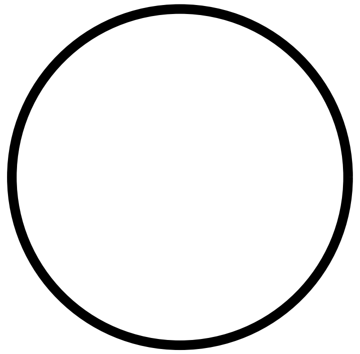
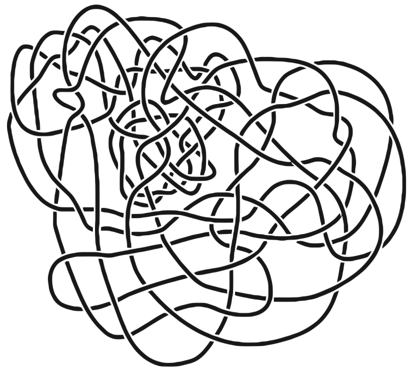
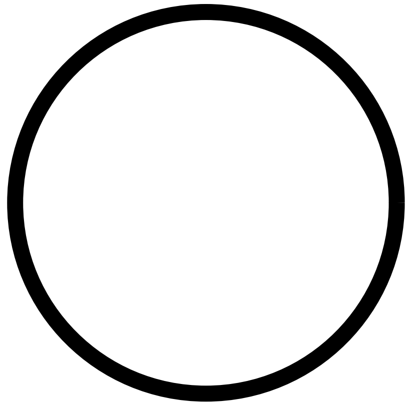
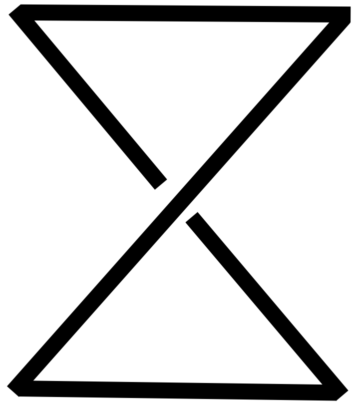
 “ = ” 	NP $\cap$ coNP
 “ = ” 	DECIDABLE

We can use **invariants** to get approximations to this problem

$$K \text{ is isotopic to } K' \implies \langle K \rangle = \langle K' \rangle$$

Knots

Polynomials give a nice list of invariants

$V_t = \Bigg($  $\Bigg) = 1$ $\qquad$ $V_t = \Bigg($  $\Bigg) = 1$

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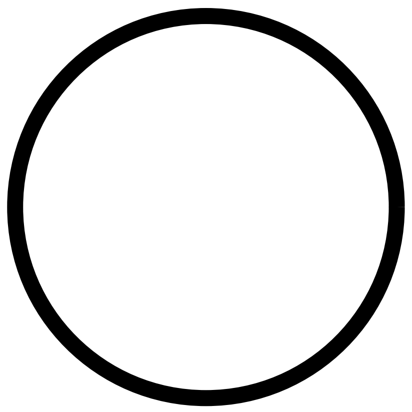
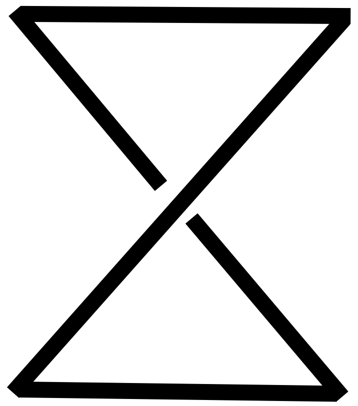
$V_t = \Big($  $\Big) = 1$ $V_t = \Big($  $\Big) = 1$

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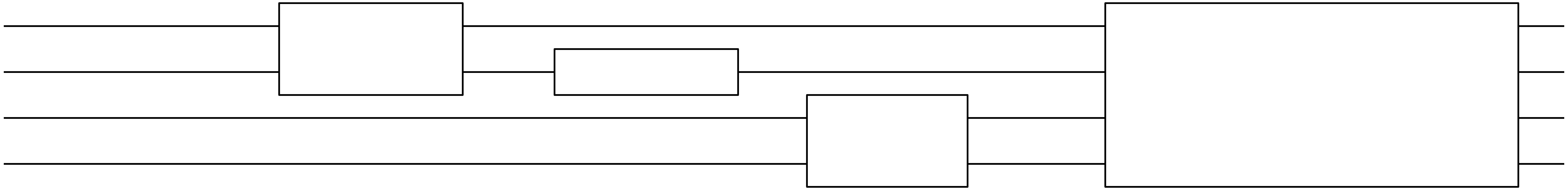
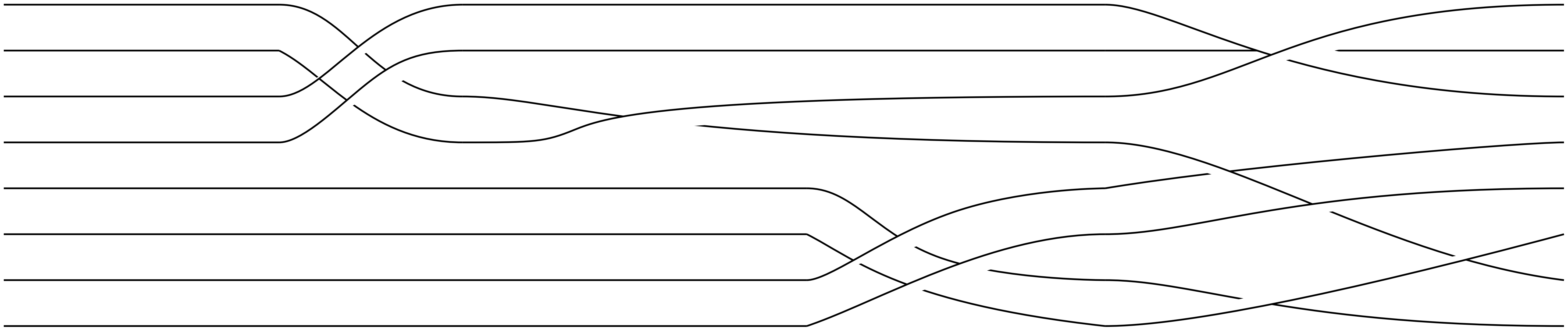
Theorem (Vertigan, Kuperberg):
Computing (or even well-approximating) the Jones polynomial at some values of t is #P-hard.

And where is quantum?





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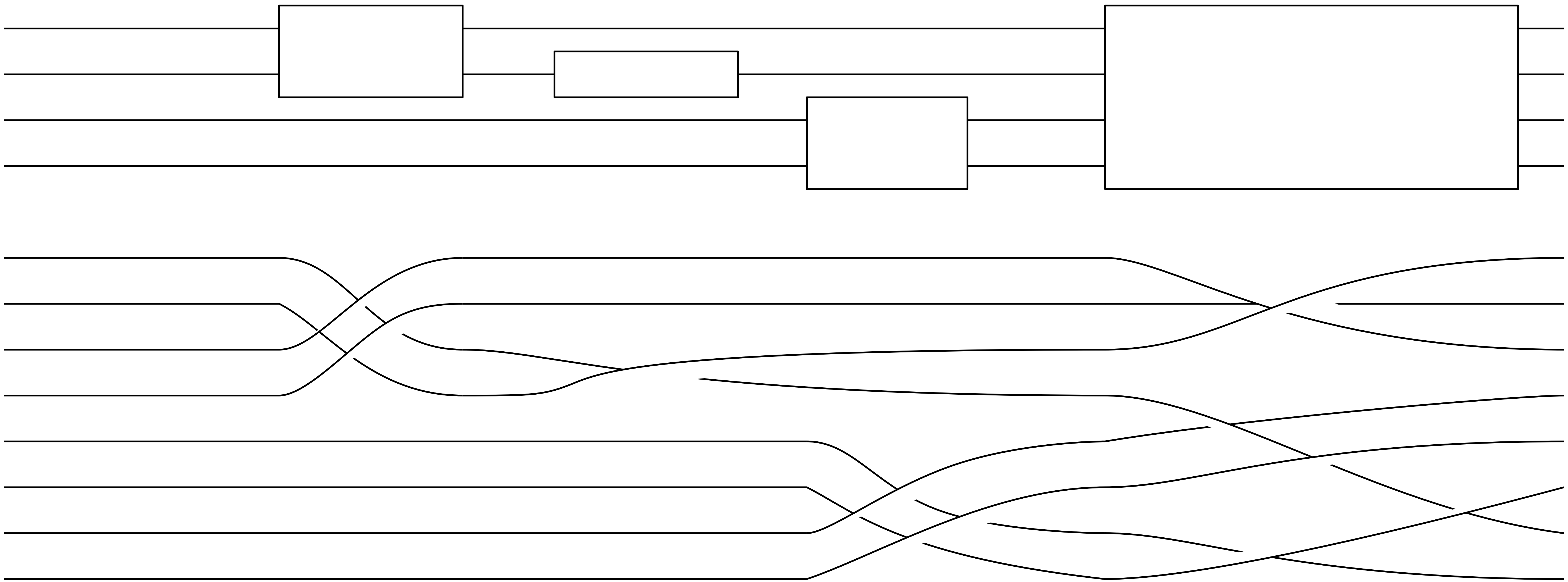

Theorem (Freedman, Larsen, Wang):
When t is certain roots of the unity, there is a dense map of the "knots" into circuits, with

$$|\langle 0^{\otimes n}|C|0^{\otimes n}\rangle|^2 \approx |V_t(L)|^2/|t^{1/2}+t^{-1/2}|^{4n}$$

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Theorem (Aaronson):

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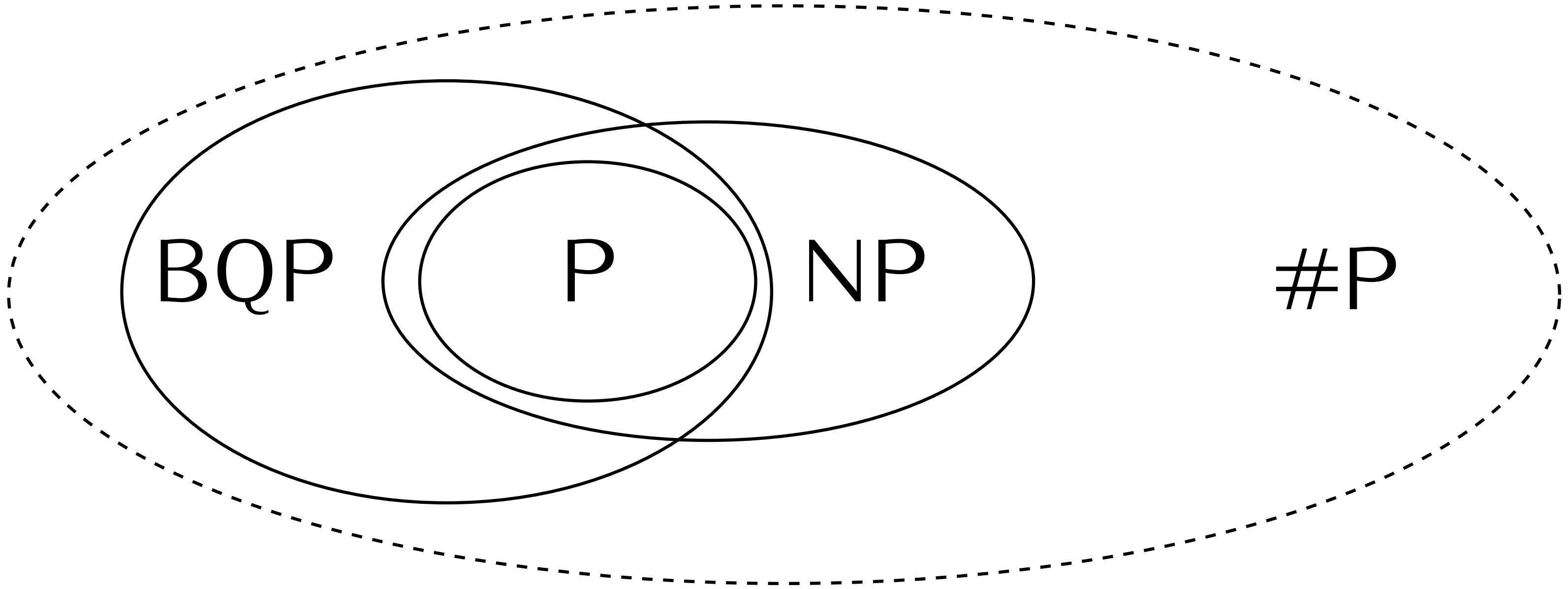
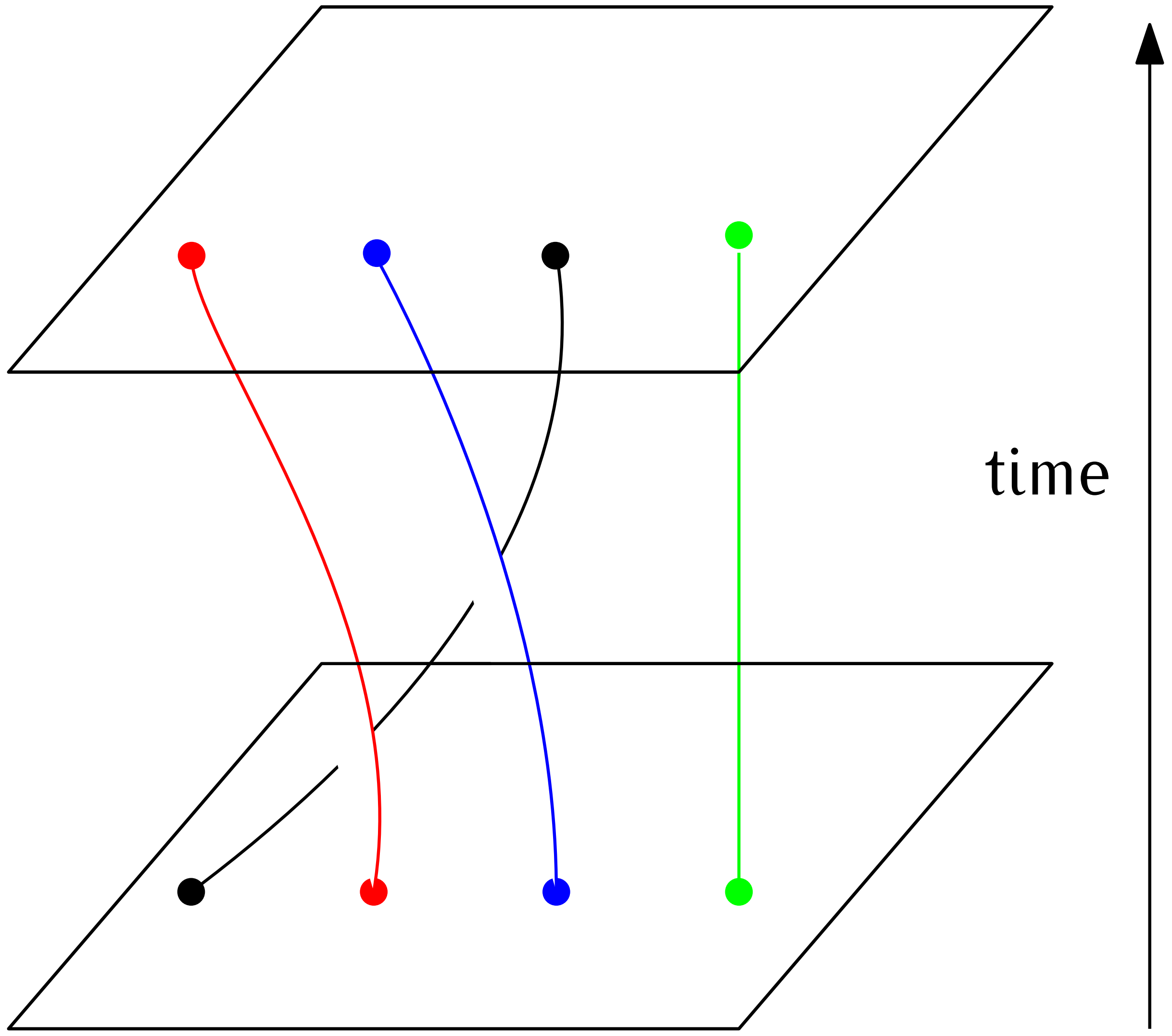
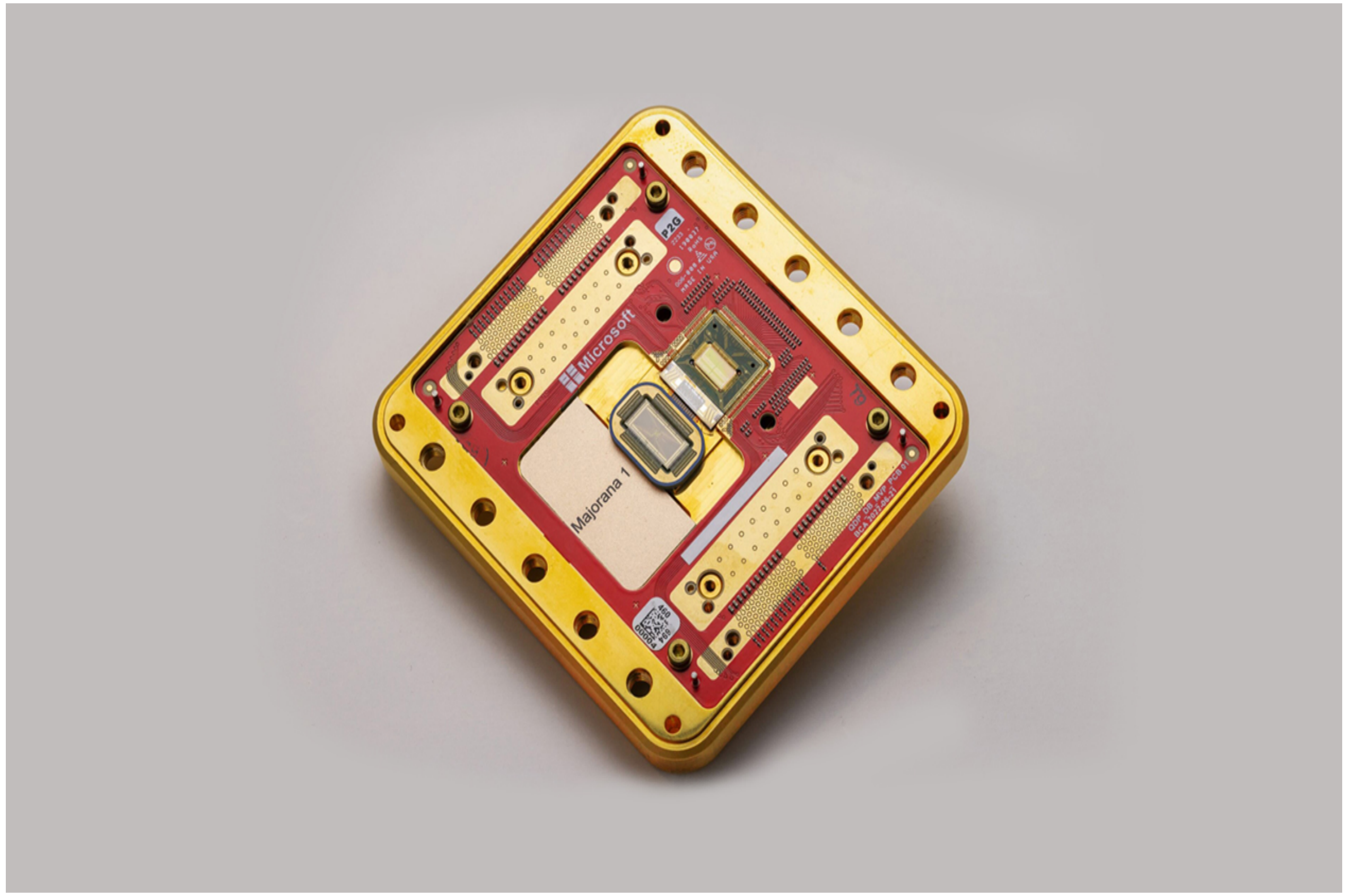


Topological quantum computing

Anyons are (quasi) particles whose behavior are very tied to Jones polynomials.





Merci!